Challenges in Modeling Extremes

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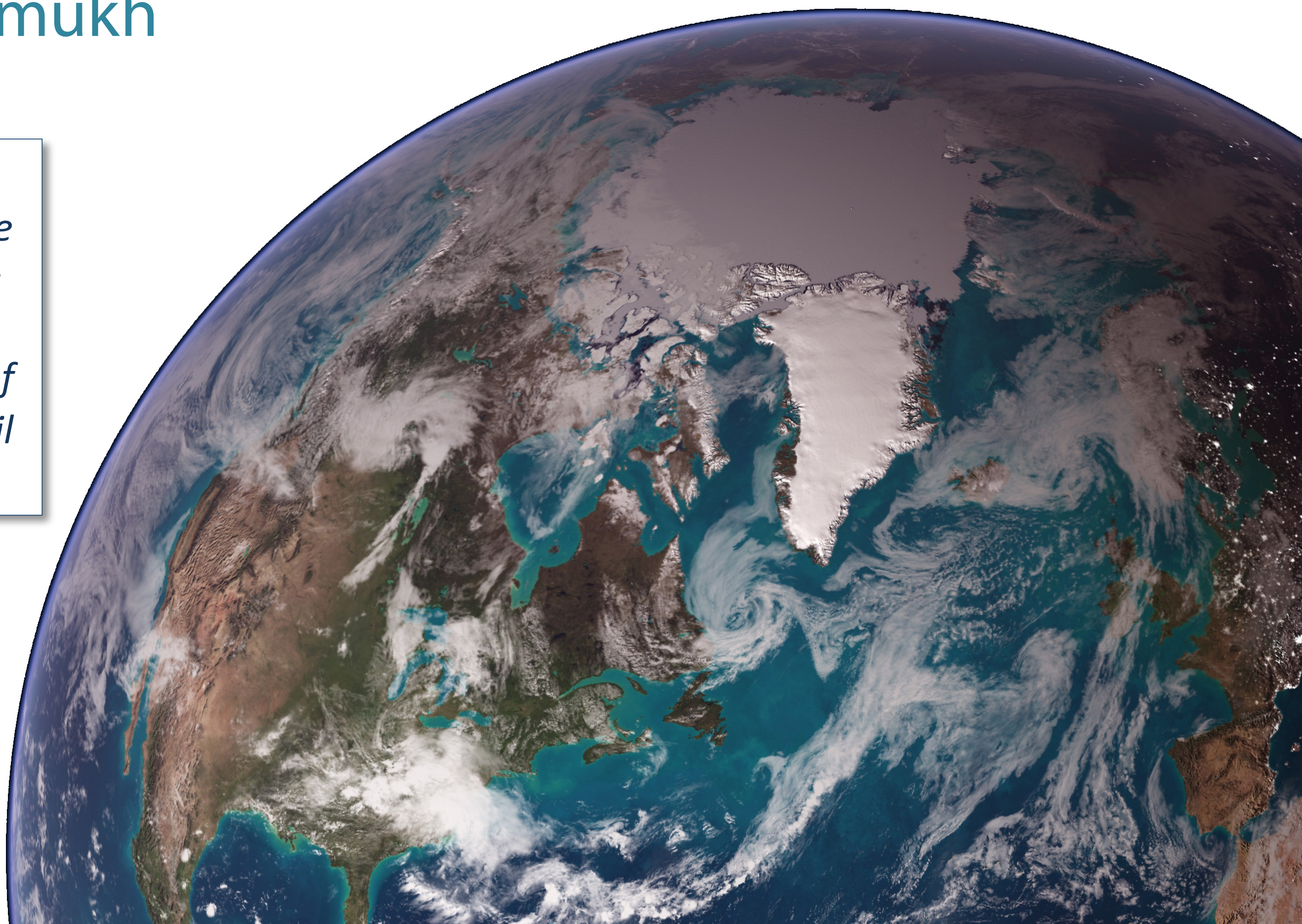
The basic challenge is to represent not just shifts of the mean but also changes in the width and shape of the probability distributions, all of which can strongly impact tail probabilities.



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Changes in extremes are not linked simply to the mean shift of the distribution

For Gaussian distributions, the sign of the change in probability of exceeding high ($x > X_T\sigma$) and low ($x < -X_T\sigma$) anomaly thresholds is given by the sign of an extremal Probability Shift Index (PSI), defined as

$$PSI(\pm) = \pm\frac{\Delta\bar{x}}{\sigma} + X_T\frac{\Delta\sigma}{\sigma} = \pm\frac{\Delta\bar{x}}{\sigma} + 2\frac{\Delta\sigma}{\sigma} \quad \text{if} \quad X_T = 2$$

where +(-) refers to the change of positive (negative extremes), X_T is the threshold in standardized units, and $\Delta\bar{x}$ and $\Delta\sigma$ are the changes in the mean and standard deviation (*Katz and Brown 1992; Sardeshmukh et al 2015*).

If there is no change in standard deviation, then a positive mean shift implies an increase of positive extremes and a decrease of negative extremes.

But if there is a change in standard deviation, then for high enough thresholds the fractional change in standard deviation dominates, and gives the same sign for changes in positive and negative extremes, regardless of the sign and magnitude of the mean shift.

The situation gets even more complicated for non-Gaussian distributions whose changes are not shape preserving.

The PDFs of daily atmospheric anomalies are generally not Gaussian.
They are distinctively skewed and heavy tailed.
This large implications for assessing, modeling, and predicting extreme anomaly risks.

Stochastically Generated Skewed distributions (SGS distributions)

$$p(x) = \frac{1}{\mathcal{N}}\left[(Ex+g)^2 + b^2\right]^{-(1+\frac{1}{E^2})} \exp\left[\frac{2g}{E^2 b}\ \arctan\left(\frac{Ex+g}{b}\right)\right]$$

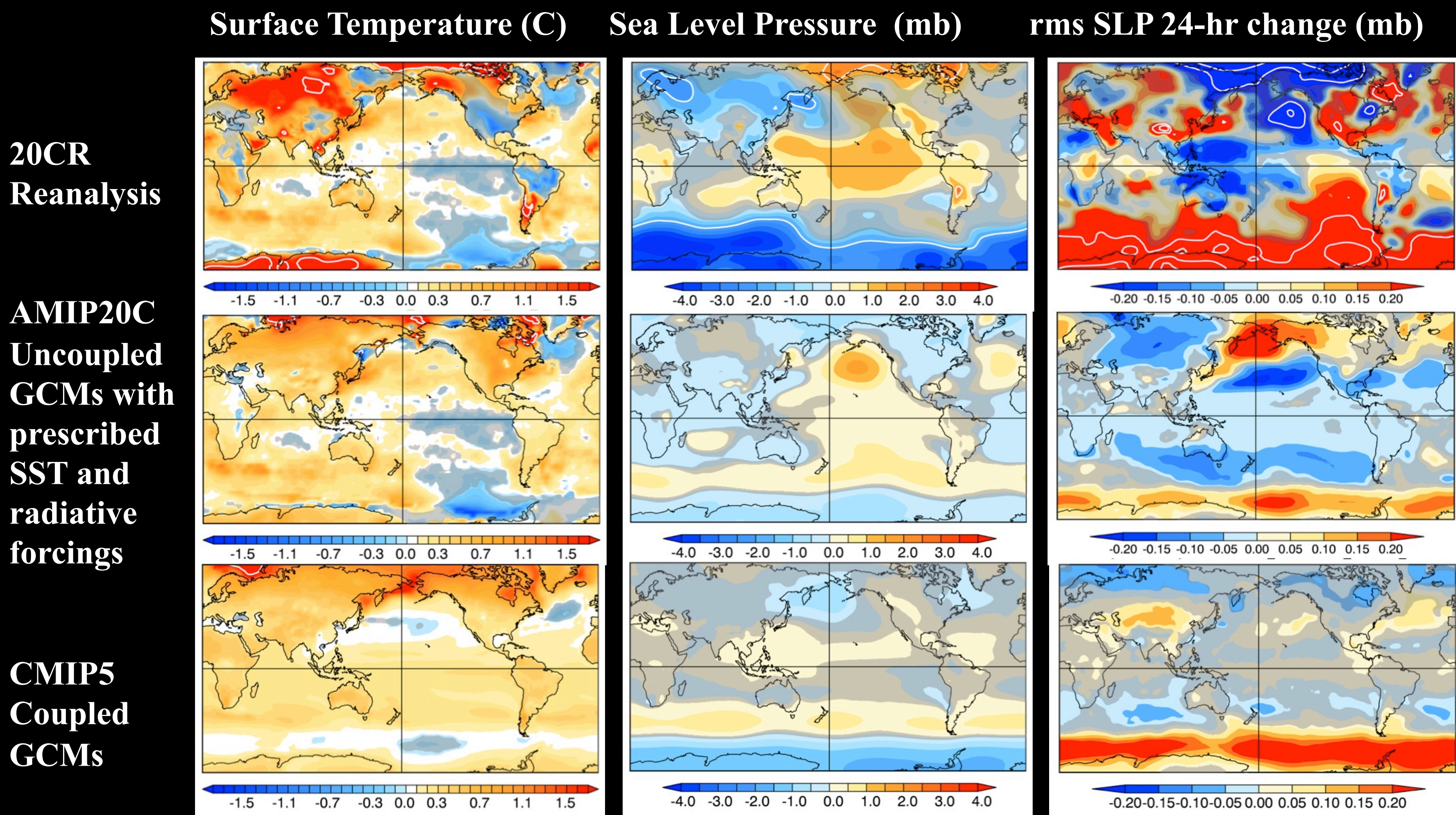
with parameters *E*, *g*, and *b*, are physically associated with linear Markov processes driven by correlated additive and multiplicative noise (CAM noise), and are particularly relevant to the analysis of weather and climate extremes.

One tail of all such distributions is heavier, and the other lighter, than a Gaussian tail beyond about 1.73 standard deviations, and its heaviness (lightness) is proportional to the Skew *S*.

(Sardeshmukh, Compo, and Penland 2015; in revision)

Atmospheric circulation and "storminess" changes associated with global warming (1943-2010 DJF average minus 1874-1942 DJF average)

Sardeshmukh, Compo, Penland, and McColl, 2015



The changes are not significant in the gray shaded areas, and there is disagreement concerning even the mean changes in many areas of "significant" change. This has important implications for assessing changes in extremes.

But what about changes in daily temperature extremes?

Are we not reasonably certain that 20th century global warming has led to increases of daily warm extremes, and decreases of daily cold extremes, in most regions ?

Isn't the mean shift (i.e. the mean warming) a useful guide at least in this context ?

We address this issue using the daily lower tropospheric 850 hPa temperature fields

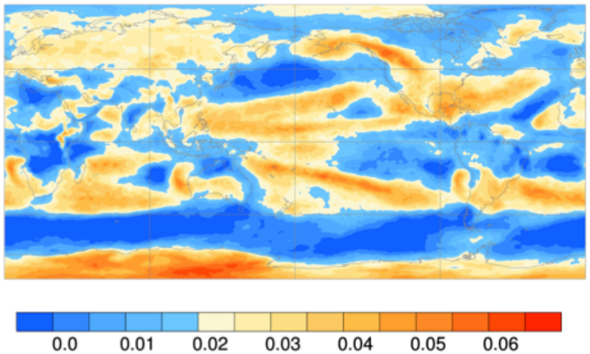
- ***In the 20th Century Reanalysis dataset ("20CR-v2c"), and***
- ***in 20th century simulations ("AMIP-v2c") generated using the same model as the reanalysis model, and with identical prescriptions of boundary and external radiative forcings.***

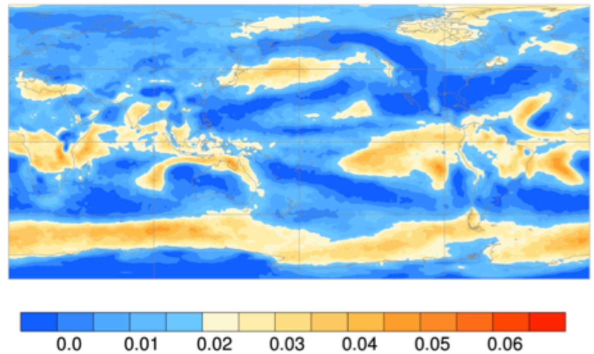
Daily 850 hPa Temperature Probabilities of exceeding ± 2 sigma (DJF 1981-2005)

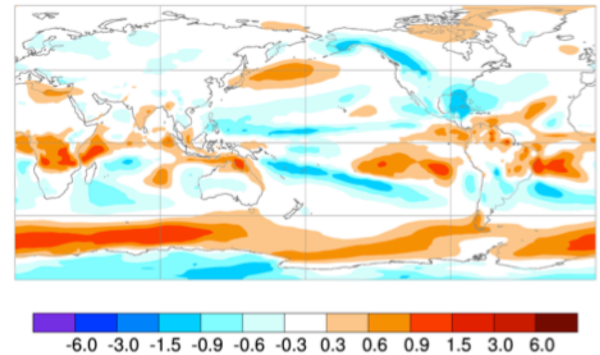
Both probabilities would be 0.022 if the distributions were Gaussian

P (x ≤ -2 sigma)	P (x ≥ 2 sigma)	Skew

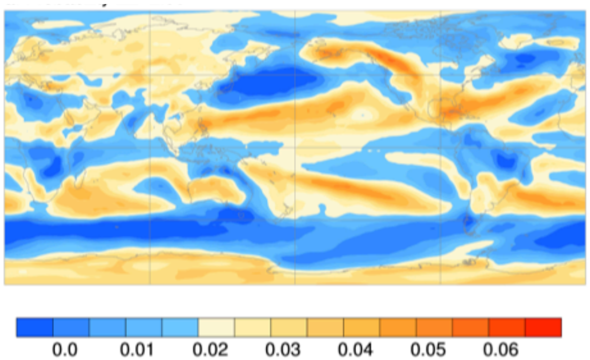
Era-Interim Reanalysis

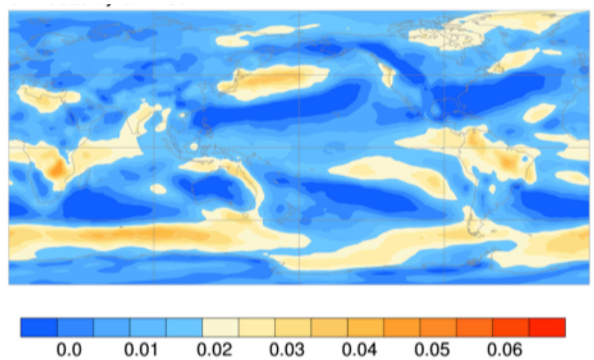


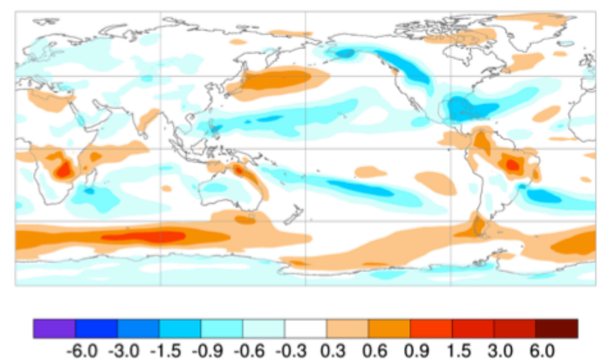




20CR-v2c Reanalysis

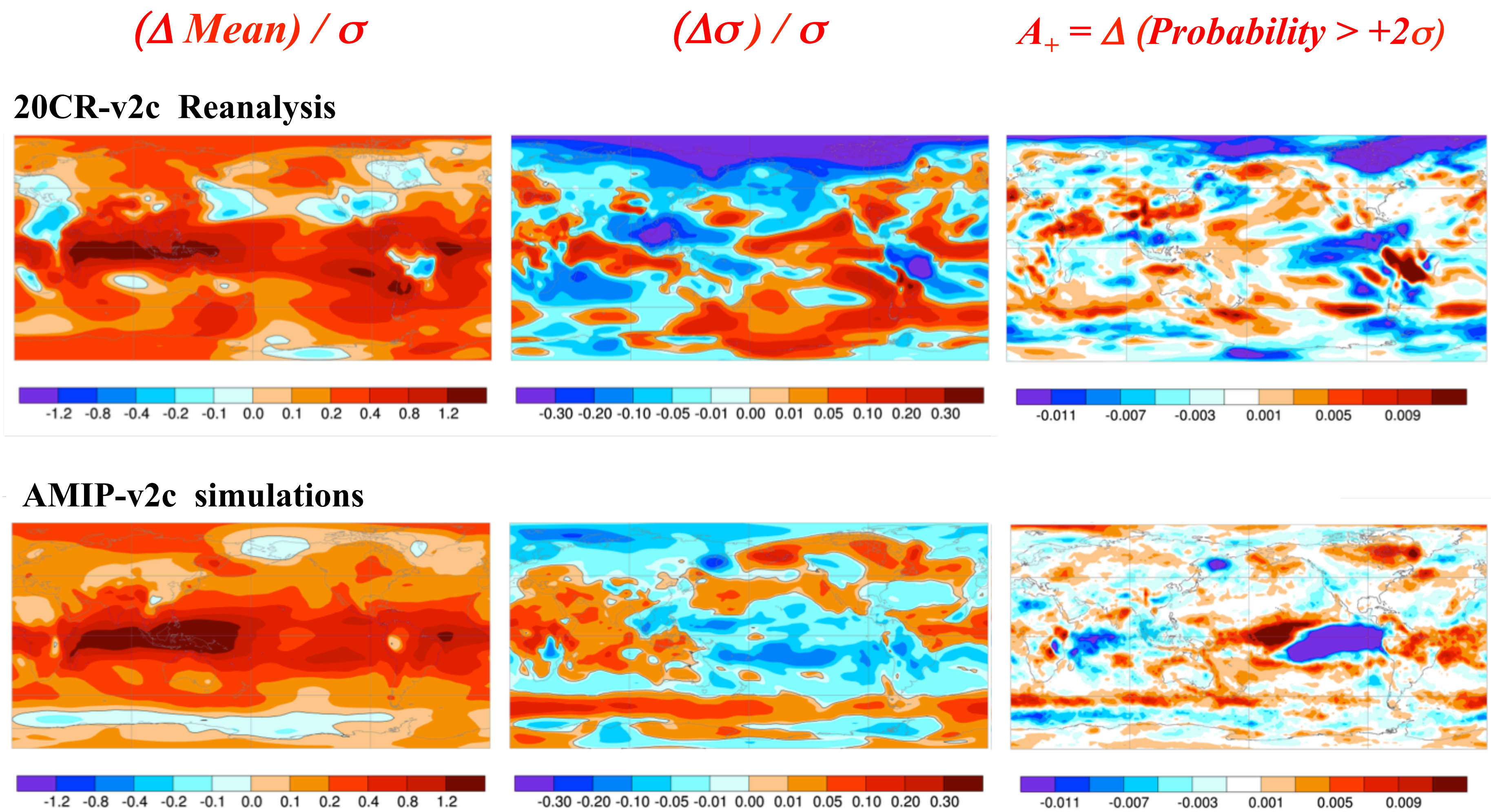






The similarity of the exceedance probability and skewness patterns is consistent with SGS theory

20th century changes in the mean, daily standard deviation, and probabilities of 850 hPa daily warm temperature extremes (DJF 1981-2005 minus DJF 1901-1925)



Note **! The pattern of the change in extreme warm daily temperature probabilities** $A_+ = \Delta\ (Probability > +2\sigma)$ **looks nothing like the pattern of mean warming.**

Summary

The fact that changes in extreme anomaly risks cannot be deduced simply from shifts of the mean is disturbing, but understandable in terms of basic weather dynamics and the Climate-Weather connection.

Climate Models must adequately represent subseasonal variability and its links with longer term changes to adequately capture the changes in the mean, width, and shape of the associated probability distributions.

Currently they have difficulty in capturing even the mean changes in many regions.

This is a challenge, but also an opportunity.